Kerr Effect

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   - Optical Kerr effect
   - Magneto-optical Kerr effect

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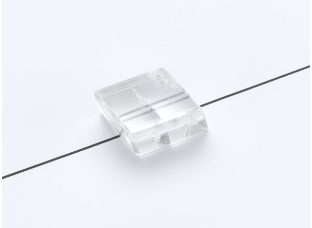
Introduction

Birefringence



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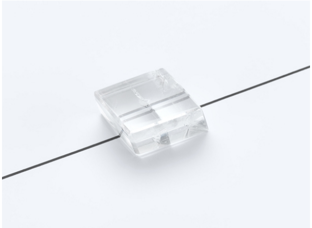

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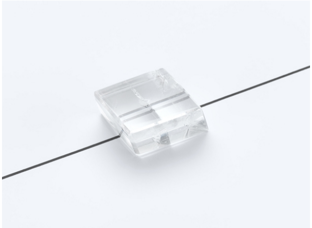

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THE
LONDON, EDINBURGH, AND DUBLIN
PHILOSOPHICAL MAGAZINE
AND
JOURNAL OF SCIENCE.

[FOURTH SERIES.]

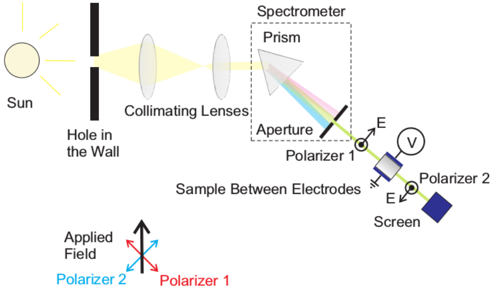
NOVEMBER 1875.

XL. *A new Relation between Electricity and Light: Dielectrified Media Birefringent. By* JOHN KERR, *LL.D., Mathematical Lecturer of the Free-Church Training College, Glasgow**.

THE thought which led me to the following inquiry was briefly this:—that if a transparent and optically isotropic insulator were subjected properly to intense electrostatic force, it should act no longer as an isotropic body upon light sent through it. Faraday was often occupied with expectations of this kind; and he has mentioned in his memoir on the Magnetization of Light, and elsewhere in his 'Researches,' how he experimented in this very direction, upon electrolytes as well as dielectrics, at different times and in many ways, but always without success†. As far as I remember, I have not read or

Introduction

Experiment set up



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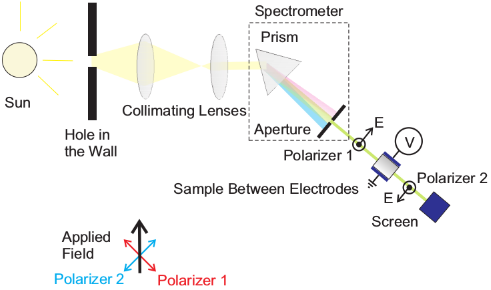

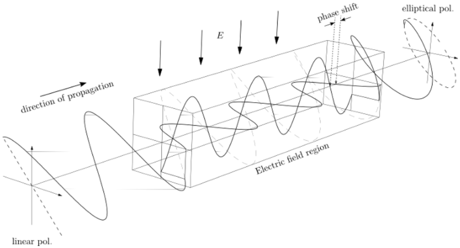

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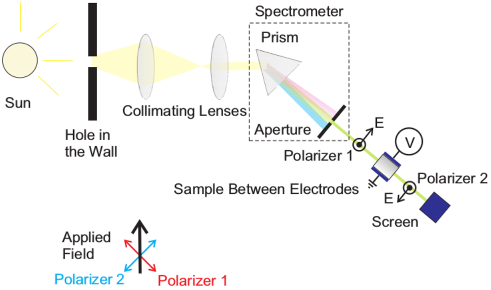
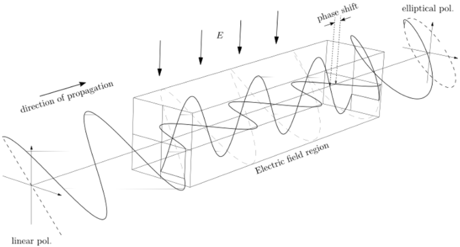
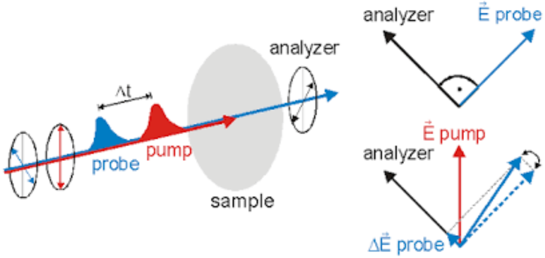




Electro-optical Kerr effect
or DC Kerr effect

Introduction

The invention of laser in 1960 provided light sources with high enough electric field strengths



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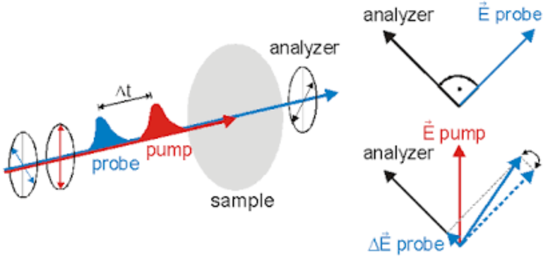

Optical Kerr effect or AC Kerr effect

Intensity-dependent refractive index

Nonlinear Optics $\rightarrow$ light-matter interactions when material's response is a non-linear function of the applied electric-field.
For a nonlinear material, the electric polarization field will depend on the electric field:

$$\mathbf{P} = \epsilon_0 \chi^{(1)} \mathbf{E} + \epsilon_0 \chi^{(2)} \mathbf{E}\mathbf{E} + \epsilon_0 \chi^{(3)} \mathbf{E}\mathbf{E}\mathbf{E} + \cdots,$$

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The i-th component for the vector **P**:

$$P_i = \varepsilon_0 \sum_{j=1}^{3} \chi_{ij}^{(1)} E_j + \varepsilon_0 \sum_{j=1}^{3} \sum_{k=1}^{3} \chi_{ijk}^{(2)} E_j E_k + \varepsilon_0 \sum_{j=1}^{3} \sum_{k=1}^{3} \sum_{l=1}^{3} \chi_{ijkl}^{(3)} E_j E_k E_l + \cdots$$

$\chi^{(n)}$: n-th order electric susceptibility

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Let's consider an electric field: $\tilde{E}(t) = E(\omega)e^{-i\omega t} + c.c$

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Kerr effect: $P(\omega) \cong \epsilon_0\chi^{(1)}E(\omega) + 3\epsilon_0\chi^{(3)}|E(\omega)|^2E(\omega) \equiv \epsilon_0\chi_{\text{eff}}E(\omega)$

Define: $\chi_{\text{eff}} = \chi^{(1)} + 3\epsilon_0\chi^{(3)}|E(\omega)|^2$.

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$$n_0 = (1 + \chi^{(1)})^{1/2}$$

$$\bar{n}_2 = \frac{3\chi^{(3)}}{2n_0}.$$

Tensor nature of the third order susceptibility

The most general third-order nonlinear process involves the interaction of waves at four different frequencies:

$$\chi_{ijkl} \equiv \chi^{(3)}_{ijkl}(\omega_4 = \omega_1 + \omega_2 + \omega_3)$$

Isotropic media: *x*, *y*, *z* axes can be chosen to make calculations as simple as possible.
Most general triclinic symmetry: $3^4 = 81$ elements, only 21 are non-zero

$$\chi_1 = \chi_{iiii} : xxxx = yyyy = zzzz \; (= xxyy + xyxy + xyyx)$$
$$\chi_2 = \chi_{jjkk} : xxyy = yyzz = zzxx = yyxx = zzyy = xxzz$$
$$\chi_3 = \chi_{jkjk} : xyxy = yzyz = zxzx = yxyx = zyzy = xzxz$$
$$\chi_4 = \chi_{jkkj} : xyyx = yzzy = zxxz = yxxy = zyyz = xzzx$$

The symmetry of a structurally isotropic medium imposes the further constraint

$$\chi_1 = \chi_2 + \chi_3 + \chi_4$$

Third order nonlinear processes

General case: applied frequencies are arbitrary $\omega_1 + \omega_2 + \omega_3 = \omega_4$. The polarization at ω_4 is given by

$$\hat{P}_i(\omega_4) = \frac{1}{2}\epsilon_0 \sum_{jkl} \chi^{(3)}_{ijkl}(\omega_4; \omega_1, \omega_2, \omega_3)\hat{E}_j(\omega_1)\hat{E}_k(\omega_2)\hat{E}_l(\omega_3),$$

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$$\hat{P}_i(\omega) = 3\epsilon_0 \sum_{jkl} \chi^{K}_{ijkl}(\omega; 0, 0, \omega)\hat{E}_j(0)\hat{E}_k(0)\hat{E}_l(\omega).$$

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Optical Kerr effect we have $\omega_1 = \omega_2 - \omega_2 + \omega_1$

$$\hat{P}_i(\omega_1) = \frac{3}{2}\epsilon_0 \sum_{jkl} \chi^{OK}_{ijkl}(\omega_1; \omega_2, -\omega_2, \omega_1)\hat{E}_j(\omega_2)\hat{E}^*_k(\omega_2)\hat{E}_l(\omega_1).$$

Electro-optical Kerr effect

Polarizations in x and y directions are

$$\begin{aligned}\hat{P}_x(\omega) &= 3\epsilon_0\chi^K_{xyyx}(\omega;0,0,\omega)E_y^2(0)\hat{E}_x(\omega)\\ &= 3\epsilon_0\chi^K_4 E_y^2(0)\hat{E}_x(\omega)\\ \hat{P}_y(\omega) &= 3\epsilon_0\chi^K_{yyyy}(\omega;0,0,\omega)E_y^2(0)\hat{E}_y(\omega)\\ &= 3\epsilon_0\chi^K_1 E_y^2(0)\hat{E}_y(\omega).\end{aligned}$$

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The DC field creates a refractive index difference between the two polarizations given by

$$n_\parallel - n_\perp \cong \frac{3(\chi^K_1 - \chi^K_4)E_y^2(0)}{2n} = \frac{3\chi^K_2 E_y^2(0)}{n},$$

The Kerr constant K of a medium is defined by

$$\Delta n \equiv n_\parallel - n_\perp = \lambda_0 K E^2(0),$$

Optical Kerr effect

A strong wave at frequency ω_2 changes the refractive index of a weak probe wave at ω_1. The operative term in the polarization is

$$\hat{P}_x(\omega_1) = \frac{3}{2}\epsilon_0 \chi^{OK}_{xxxx}(\omega_1;\ \omega_2, -\omega_2, \omega_1)|\hat{E}_x(\omega_2)|^2 \hat{E}_x(\omega_1),$$

which implies that the refractive index of the weak wave is changed by

$$\Delta n_x \cong \frac{3\chi^{OK}_{xxxx} I(\omega_2)}{2n(\omega_1)n(\omega_2)c\epsilon_0}.$$

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An special case of the optical Kerr effect occurs when a single beam at $\omega = \omega_1 = \omega_2$ modifies its own refractive index

$$\hat{P}_x(\omega) = \frac{3}{4}\epsilon_0 \chi^{OK}_1(\omega;\ \omega, -\omega, \omega)|\hat{E}_x(\omega)|^2 \hat{E}_x(\omega).$$

This implies that the refractive index is changed to

$$n = n_0 + \left(\frac{3\chi^{OK}_1}{4n_0^2 c\epsilon_0}\right) I = n_0 + n_2 I,$$

Applications

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* Spectroscopy of liquids
* Waveplates
* Photonic and electro-optic devices

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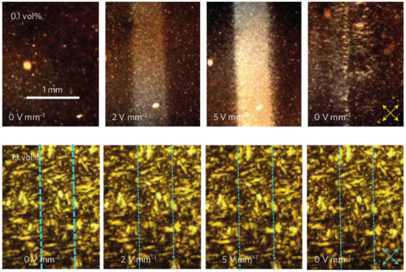
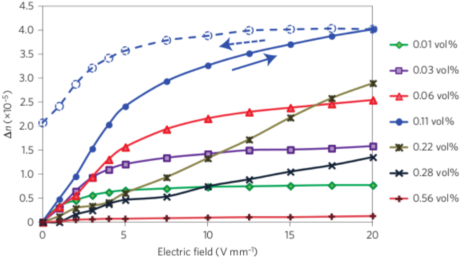
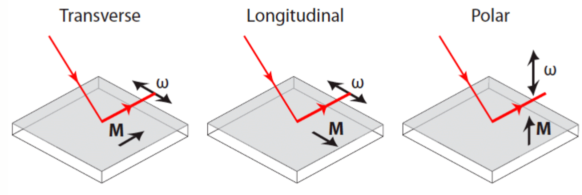




Field-induced birefringence was generated by applying electric fields (10 kHz) to an aqueous 0.1 vol% graphene oxide dispersion. In the same cell structure with a 1.1 vol% GO, no change was detected up to 20 V mm^{-1}. *Shen (Nat. Mat. 2014).*

Magneto-optical Kerr effect

Magneto-optical Kerr effect (MOKE): light reflected from a magnetized material has a slightly rotated plane of polarization.



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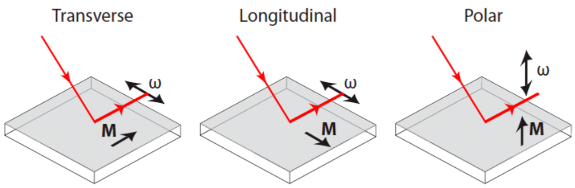

It is used in materials science research in devices such as the Kerr microscope, to investigate the magnetization structure of materials.
Thanks to its high accuracy, high temporal and spatial resolution and very fast response, the MOKE is a powerful method to study the magnetic properties of ultrathin and multilayer films.

Conclusions

* A study of different types of electro-optical effects has been presented, trying to understand the behavior of the interaction of light with matter when the response of the medium is a non-linear function of the applied electric or magnetic field.
* It has been explored in some properties of the electric susceptibility tensor, necessary to explain the non-linear effects, considering symmetries in isotropic media.
* The electro-optical, optical and magneto-optical Kerr effect have many powerful applications that are already being carried out in research.

References

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Obrigada

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